Materials Selection in Mechanical Design by M.F.Ashby

Excercises & Solutions Unit 2

J. Michler

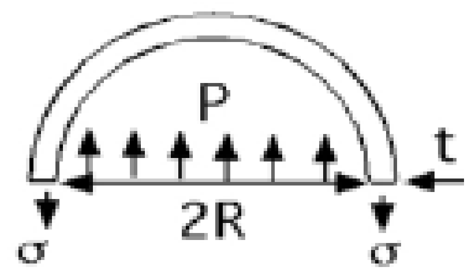
Materials for pressure vessel

Materials for pressure vessel

Specification

Function	Contain pressure p
Objectives	Minimum thickness Minimum weight Maximise pressure Maximise allowable crack length
Constraints	• Yield before break or • Must leak before break • Toughness adequate • Diameter $2R$ and pressure Δp specified
Free variables	• Wall thickness t • Material

Pressure vessels are pressured-limited, minimum weight, designs

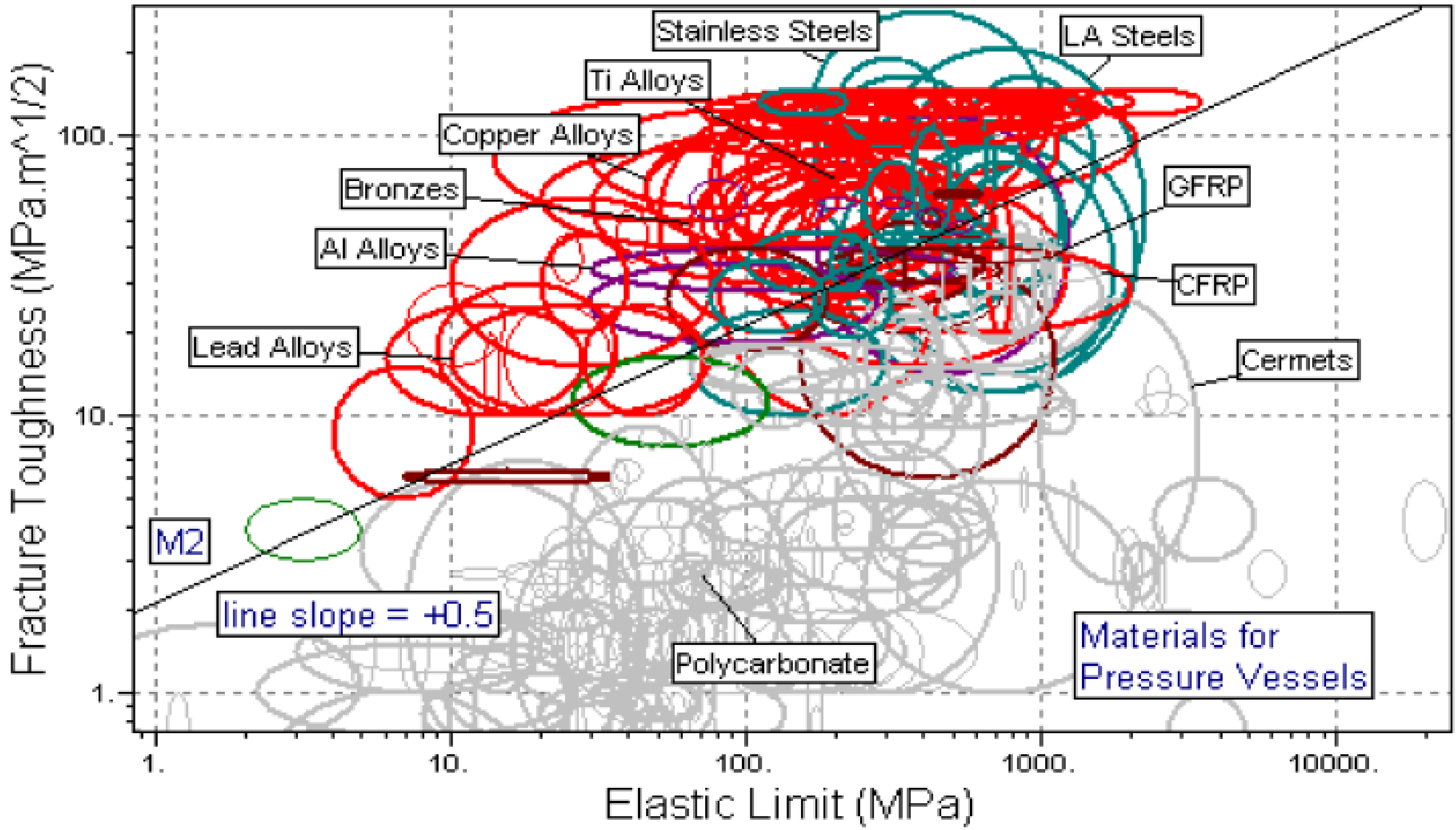


We idealize the pressure vessel as a thin walled sphere

Summary pressure vessels

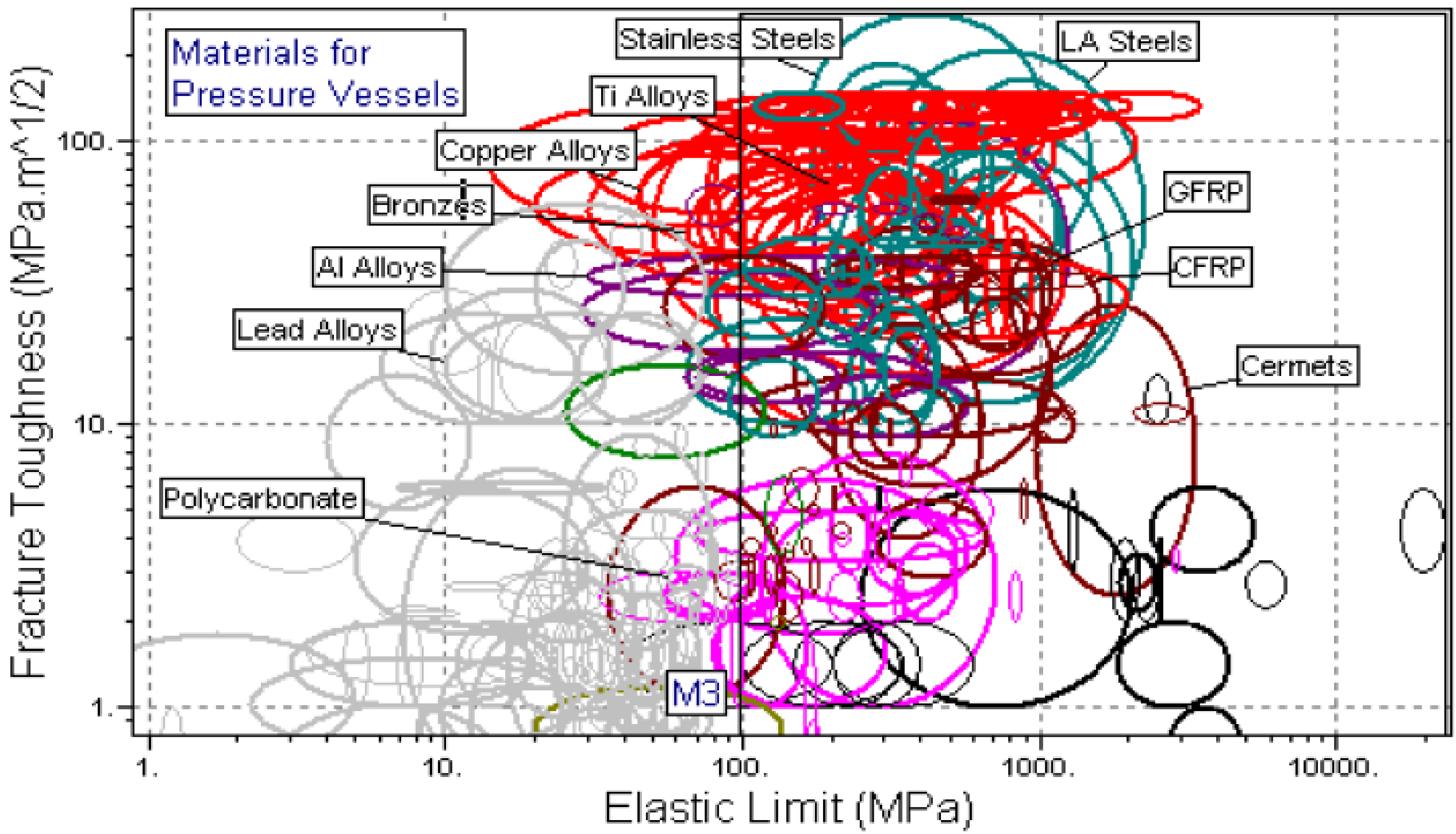
Objectives:	**Index**
Maximize pressure, for a given maximum crack size	$M_1 = K_{IC}$
Maximize safety using leak-before-break (Objective maximise pressure under constraint leak before break)	$M_2 = \frac{K^2{}_{IC}}{\sigma_y}$
Maximize safety using yield-before-break (Objective maximise size of safe crack)	$M_3 = \frac{K_{IC}}{\sigma_y}$
Minimize thickness	$M_4 = \sigma_y$
Minimize weight	$M_5 = \frac{\rho}{\sigma_y}$

Solutions - Materials for pressure vessel



Index M_2

Solutions - Materials for pressure vessel



Index M_4 **= 100 MPa**

Solutions - Materials for pressure vessel: postscript

In practice large pressure vessels are always made of steel. Those for models — for instance a model steam engine — are copper. Copper is favoured in the small scale application because of its greater resistance to corrosion. When weight is important, copper alloys are not a good choice; aluminium alloys, GFRP and CFRP offer the best combination of toughness, strength and low density.

Exercise Multiple objectives: strength at min. weight vs. price

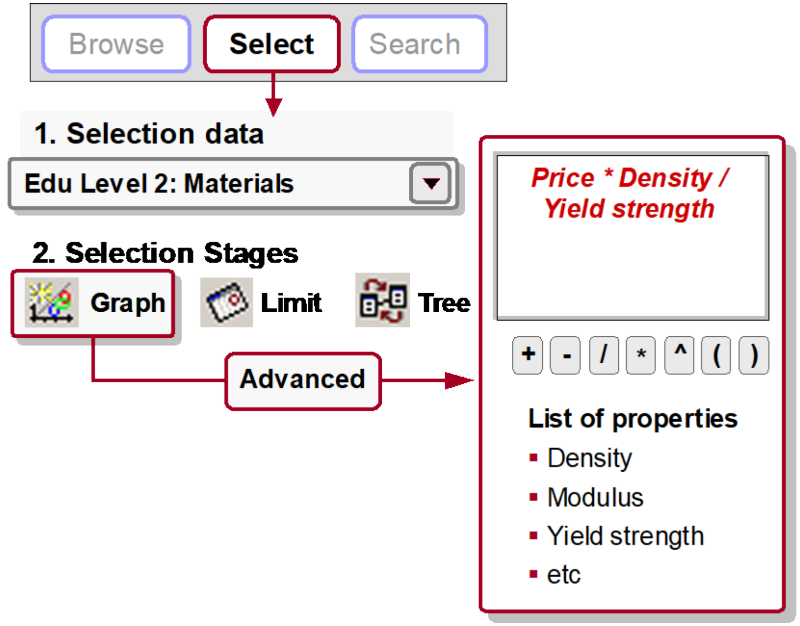
Explore the trade-off between minimizing **mass** and **price** for a component loaded in tension. The lightest material (that meets all other constraints, of course) is that with the lowest value of

$$\rho / \sigma_y$$

(density divided by yield strength). The one with lowest price is that with the lowest value of

$$C_m \rho / \sigma_y$$

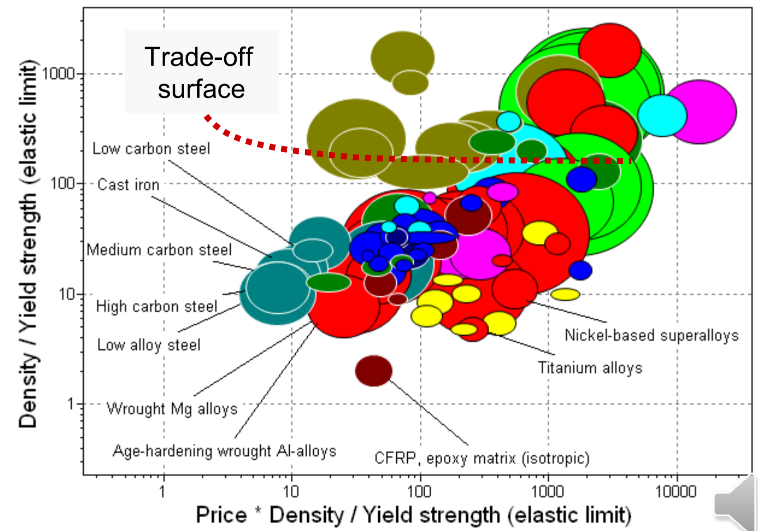
(the same as above, multiplied by the price per unit mass)



Graph stage using the Advanced facility

- X-axis: Density / Yield strength
- Y-axis: Price * Density / Yield strength

(no need to plot trade -off surface)



Example Multiple Objectives: Freezers (1)

Insulating walls for freezers

Freezers and refrigerated trucks have panelwalls that provide thermal insulation, and at the same time are stiff, strong and light (stiffness to suppress vibration, strength to tolerate rough usage). To achieve this the panels are usually of sandwich construction, with two skins of steel, aluminum or GFRP (providing the strength) separated by, and bonded to, a low density insulating core. In choosing the core we seek to minimize thermal conductivity, λ, and at the same time to maximize stiffness, because this allows thinner steel faces, and thus a lighter panel, while still maintaining the overall panel stiffness. The table summarizes the design requirements.

Function	• *Foam for panel-wall insulation*
Constraint	• *Panel wall thickness specified.*
Objectives	• *Minimize foam thermal conductivity, λ* • *Maximize foam stiffness, meaning Young's modulus, E*
Free variables	• *Choice of material*

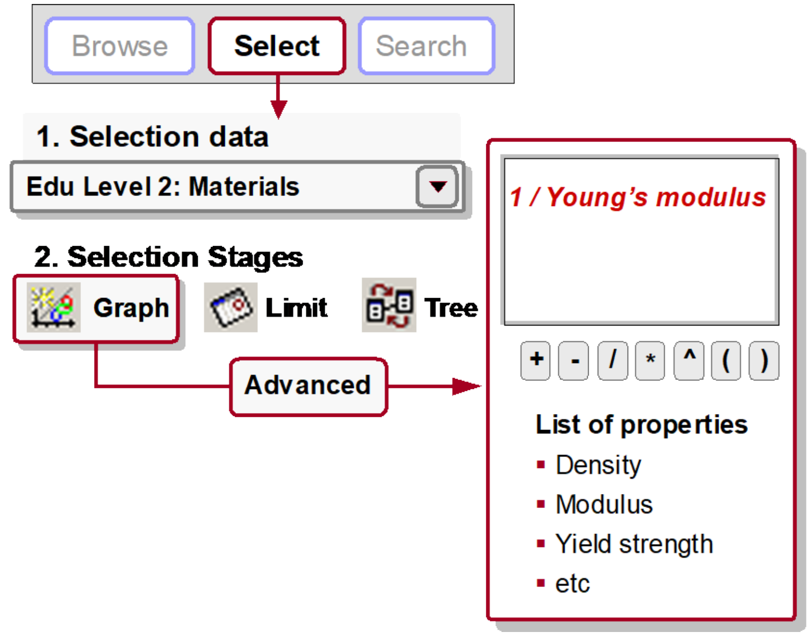
Example Multiple Objectives: Freezers (2)

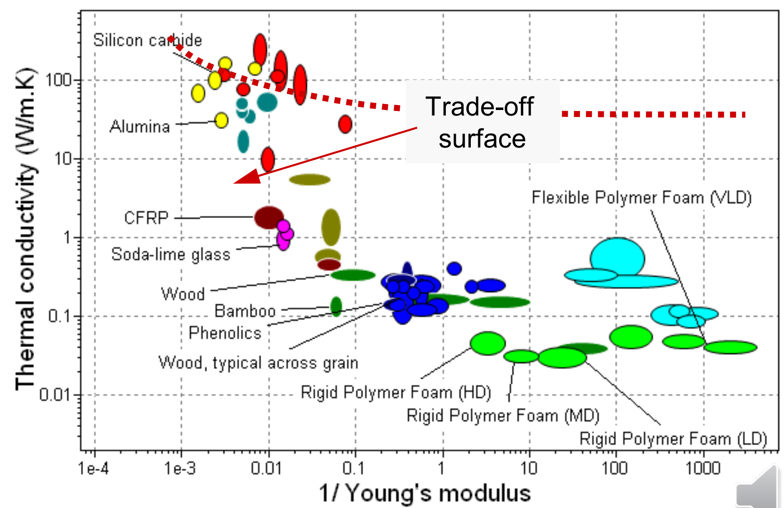
Freezers have panel-walls that provide thermal insulation and at the same time are stiff and strong. They are of sandwich construction with two skins of steel separated and bonded to an insulating core. In choosing the core we seek to minimize thermal conductivity while at the same time maximizing stiffness (and so Young's modulus), allowing thinner, and thus lighter and cheaper faces. Make an appropriate trade-off plot to find materials that best do both. Remember that both objectives must be minimized – so use the reciprocal of Young's modulus.

Start with Level 2, then **import** Level 3

Graph stage

- X-axis: 1/Young's modulus (use the Advanced facility)
- Y-axis: Thermal conductivity
- No need to plot trade off surface (You could copy, paste into WORD or Adobe Illustrator and sketch in a trade-off surface)





Example Multiple Objectives: Freezers (3)

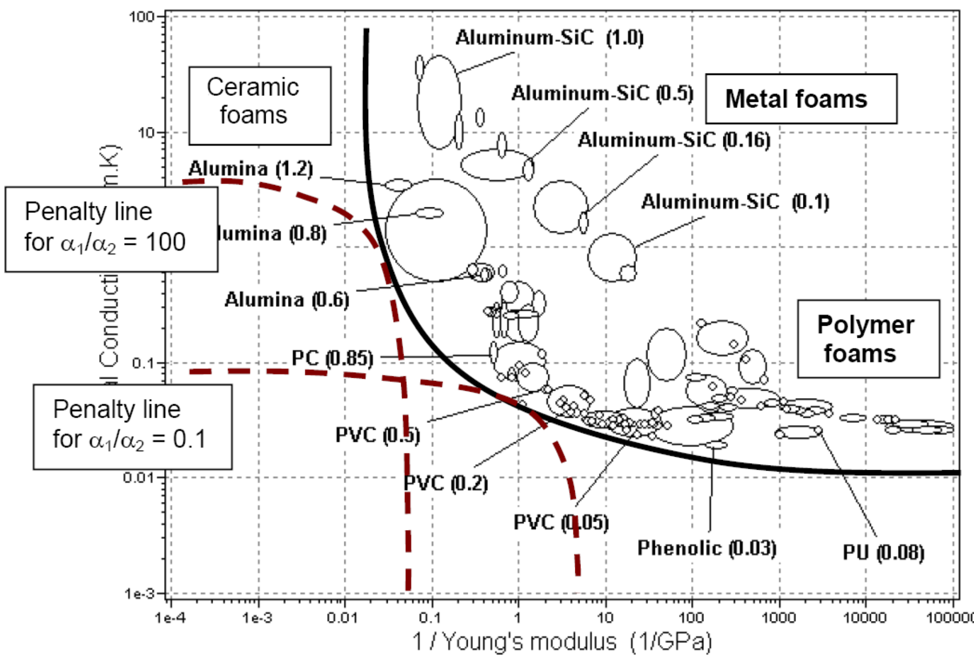
(2) If it is desired to go further, it is necessary to construct a penalty function:

$$Z = \alpha_1 \lambda + \alpha_2 \left(\frac{1}{E} \right)$$

Z is to be minimized, so α_1 is a measure of the value associated with reducing heat flow; α_2 a measure of the value associated with reducing core compliance. Rearranging the equations gives

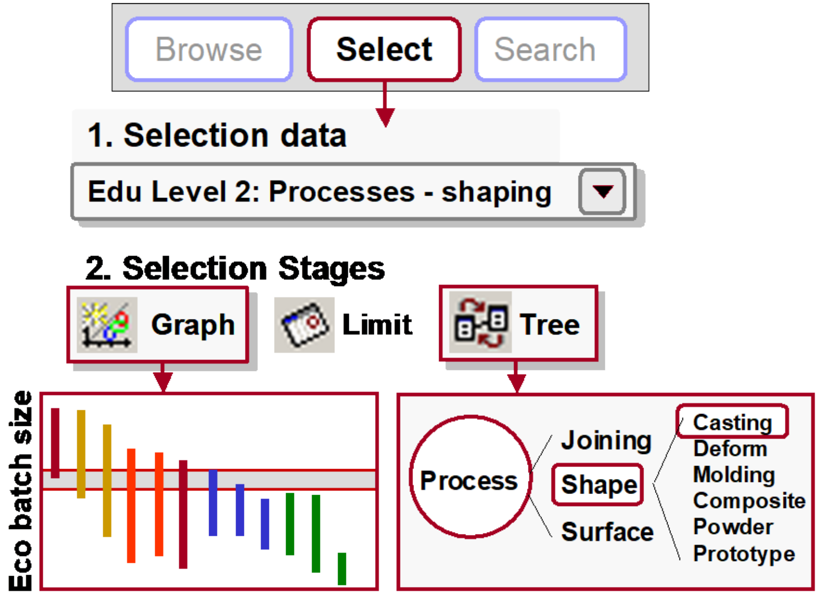
$$\lambda = \frac{Z}{\alpha_1} - \frac{\alpha_2}{\alpha_1} \left(\frac{1}{E} \right)$$

If the axes were linear, this equation would be that of a family of straight, parallel, lines on the λ vs. $1/E$ diagram, of slope $-\alpha_2 / \alpha_1$, each line corresponding to a value of Z / α_1. In fact the scales are logarithmic, and that leads instead to a set of curved lines. One such line is sketched below for values $\alpha_2 / \alpha_1 = 0.01$ (meaning that thermal insulation is considered very important, and stiffness less important) and for $\alpha_2 / \alpha_1 = 100$ (meaning the opposite). The foam nearest the point at which the penalty lines are tangent to the trade-off surface is the best choice. In the first example PVC foam with a density of about 0.1 Mg/m^3 is the best choice, but in the second a ceramic or even a metal foam is a better choice.



Exercise: using economic batch size

Metals can be cast in many different ways. A metal is to be cast to a simple shape. It is expected that between 20,000 and 30,000 units will be needed. Use Level 3 of CES the software to identify the subset of casting processes that are economic at this batch size.



- Change the database to **CES Edu Level 3**
- Use the Selection data tab to choose

  ProcessUniverse: All Processes

- Apply a **Tree Stage** to isolate **Shaping \ Casting**
- Make a **Graph stage** with **Economic batch size** on the y-axis
- Use a Box selection to apply the limits

Possible results:

Cosworth casting

Ferro die casting

Gravity die casting

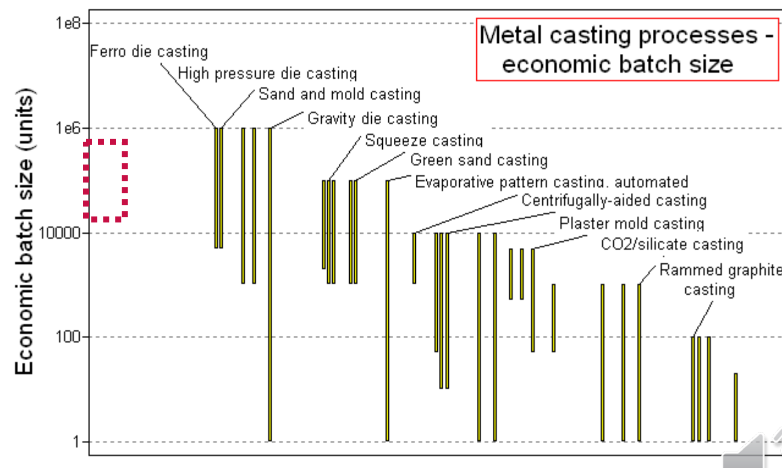
Green sand casting, automated

High pressure die casting

Low pressure die casting

Shell casting

Squeeze casting



Exercise: using the cost model

A simple shape is to cast in aluminum alloy. It is suggested that **gravity die casting** and **ceramic mould** casting might be good choices of process. Plot the cost against batch size for these processes using Level 3 of the database. Check under "parameters "material cost" and "compontent mass", "overhead rate", "capital write-off time" and "load factor". Which process is cheaper for a batch size of 1000? Assume that as the piston is simple in shape, it will fall near the bottom of each cost-band.

Result. Gravity die casting is the less expensive process for a batch size of 1000.